Name ____ Test 3, Spring 2021

1) Find all the eigenspaces of the matrix below (15 points)

$$\begin{bmatrix} -1 & -3 & 3 \\ 0 & -1 & 0 \\ 0 & -3 & 2 \end{bmatrix}$$

2) Find the diagonalizaion of the matrix from the previous problem. (5 points)

(If you couldn't solve the previous problem, make up an answer to answer this problem)

3) Given the basis below, find an orthogonal basis for the same vector space. (10 points)

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \right\}$$

4) Answer the following questions. (3 points each)

A) Let A be a 5×5 with eigenvalues $0, 0, 1, 2, 3$. What is the maximum rank of A?

B) Let A be a 3×5 matrix whose nullity is 4. When row reduced, how many rows of zeroes are there?

C) Consider a system of 4 equations and 4 variables that has a unique solution. When row reduced, how many pivots does the corresponding matrix have?

D) Let A be a 6×6 matrix whose corresponding linear transformation T is onto. Is T one-to-one?

E) Let A be a 3×3 matrix whose corresponding linear transformation T is not one-to-one. What is the determinant of A?

5) Given the two bases and linear transformation below, draw the diagram that represents this information. (10 points)

$$B_1 = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \end{bmatrix} \right\}, B_2 = \left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \end{bmatrix} \right\}, T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{B_1} \right) = \begin{bmatrix} x_2 \\ 3x_1 - x_2 \end{bmatrix}_{B_2}$$

6) Find the product below. (10 points)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$$

7) Reduce the matrix below to reduced echelon form. (10 points)

$$\begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 6 \\ 0 & 1 & 3 & 4 & 1 \end{bmatrix}$$

8) Find the product below. (5 points)

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 & 2 & 2 \\ 3 & 1 & 3 & 3 & 3 \\ 4 & 4 & 1 & 4 & 4 \\ 5 & 5 & 5 & 1 & 5 \\ 6 & 6 & 6 & 6 & 1 \end{bmatrix}$$

9) Find the length of the vector below. (5 points)

$$\begin{bmatrix} 3 \\ 4 \\ 0 \\ 12 \end{bmatrix}$$

10) Given system of equations below, write the corresponding matrix equation. (5 points)

$$x + 2y = 3$$
$$4x + 5y = 6$$

11) Find a formula for the pseudoinverse of the matrix below. (5 points)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

12) Given the information below about three bases B_1, B_2, B_3, and two linear transformaitons T_1 and T_2, find a formula for $[T_1]_{B_1}^{B_2}$. (5 points)

$$B_1 = \left\{\begin{bmatrix}2\\0\end{bmatrix}, \begin{bmatrix}0\\3\end{bmatrix}\right\}$$

$$B_2 = \left\{\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}4\\0\end{bmatrix}\right\}$$

$$B_3 = \left\{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}3\\4\end{bmatrix}\right\}$$

$$[T_2 \circ T_1]_S^S = \left\{\begin{bmatrix}5\\6\end{bmatrix}, \begin{bmatrix}2\\2\end{bmatrix}\right\}$$

$$[T_2]_{B_2}^{B_3} = \left\{\begin{bmatrix}3\\1\end{bmatrix}, \begin{bmatrix}4\\2\end{bmatrix}\right\}$$